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MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Tuesday 21 May 2013 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 10]*

The random variable X is normally distributed with unknown mean μ and unknown variance σ^2. A random sample of 20 observations on X gave the following results.

$$\sum x = 280,\ \sum x^2 = 3977.57$$

(a) Find unbiased estimates of μ and σ^2. *[3 marks]*

(b) Determine a 95 % confidence interval for μ. *[3 marks]*

(c) Given the hypotheses

$$\mathrm{H}_0 : \mu = 15;\ \mathrm{H}_1 : \mu \neq 15\,,$$

find the p-value of the above results and state your conclusion at the 1 % significance level. *[4 marks]*

2. *[Maximum mark: 12]*

A hockey team played 60 matches last season. The manager believes that the number of goals scored by the team in a match could be modelled by a Poisson distribution and he produces the following table based on the season's results.

Number of goals	0	1	2	3	4	5
Frequency	8	9	17	14	7	5

(a) State suitable hypotheses to test the manager's belief. *[1 mark]*

(b) The manager decides to carry out an appropriate χ^2 goodness of fit test.

(i) Construct a table of appropriate expected frequencies correct to **four decimal places**.

(ii) Determine the value of χ^2_{calc} and the corresponding p-value.

(iii) State whether or not your analysis supports the manager's belief. *[11 marks]*

3. *[Maximum mark: 9]*

The number of machine breakdowns occurring in a day in a certain factory may be assumed to follow a Poisson distribution with mean μ. The value of μ is known, from past experience, to be 1.2. In an attempt to reduce the value of μ, all the machines are fitted with new control units. To investigate whether or not this reduces the value of μ, the total number of breakdowns, x, occurring during a 30-day period following the installation of these new units is recorded.

(a) State suitable hypotheses for this investigation. *[1 mark]*

(b) It is decided to define the critical region by $x \le 25$.

(i) Calculate the significance level.

(ii) Assuming that the value of μ was actually reduced to 0.75, determine the probability of a Type II error. *[8 marks]*

4. *[Maximum mark: 14]*

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \dfrac{3x^2 + 2x}{10}, & \text{for } 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}.$$

(a) (i) Determine an expression for $F(x)$, valid for $1 \le x \le 2$, where F denotes the cumulative distribution function of X.

(ii) Hence, or otherwise, determine the median of X. *[6 marks]*

(b) (i) State the central limit theorem.

(ii) A random sample of 150 observations is taken from the distribution of X and $\bar{X}$ denotes the sample mean. Use the central limit theorem to find, approximately, the probability that $\bar{X}$ is greater than 1.6. *[8 marks]*

5. *[Maximum mark: 15]*

When Ben shoots an arrow, he hits the target with probability 0.4. Successive shots are independent.

(a) Find the probability that

(i) he hits the target exactly 4 times in his first 8 shots;

(ii) he hits the target for the 4^{th} time with his 8^{th} shot. *[6 marks]*

(b) Ben hits the target for the 10^{th} time with his X^{th} shot.

(i) Determine the expected value of the random variable X.

(ii) Write down an expression for $\mathrm{P}(X = x)$ and show that

$$\frac{\mathrm{P}(X = x)}{\mathrm{P}(X = x - 1)} = \frac{3(x-1)}{5(x-10)}.$$

(iii) Hence, or otherwise, find the most likely value of X. *[9 marks]*